Potential models for radiative rare B decays

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We compute the branching ratios for the radiative rare decays of B into K-meson states and compare them to the experimentally determined branching ratios for inclusive decay $b \to s\gamma$ using the nonrelativistic quark model, and form factor definitions consistent with the HQET covariant trace formalism. Such calculations necessarily involve a potential model. In order to test the sensitivity of the calculations to potential models we use three different potentials: namely, the linear potential, screening confining potential, and heavy quark potential as it stands in QCD. We find the branching ratios relative to the inclusive $b \to s\gamma$ decay to be $(16.07 \pm 5.2)\%$, $(19.75 \pm 5.3)\%$, and $(11.18 \pm 4.6)\%$ for $B \to K^{\star}(892)\gamma$ for the linear, screening confining, and heavy quark potential, respectively, while the corresponding values of branching ratios for $B \to K_2^{\star}(1430)\gamma$ relative to $B \to K^{\star}(892)\gamma$ are 0.45 ± 0.13, 0.24 ± 0.06, and 0.46 ± 0.14, respectively. All these values are consistent with the corresponding present CLEO experimental values: $(16.25 \pm 1.21)\%$ [for $B \to K^{\star}(892)\gamma$] and $0.39^{+0.15}_{-0.13}$ [for $B \to K_2^{\star}(1430)\gamma$].

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I. INTRODUCTION

The flavor changing weak decays of mesons have always been a rich source of information about basic interactions in particle physics. In particular, the radiative B decays $B \to K^{**}\gamma$[$K^{**} \sim K^*(892)$, $K^*(1430)$ etc.] have received intensive theoretical studies. The presence of a heavy b quark permits the use of heavy quark effective theory (HQET) in evaluating the relevant hadronic matrix elements where the relevance of the use of a potential model comes in. One purpose of our paper is to test the sensitivity of the branching ratios for $B \to K^{**}\gamma$ decays relative to $B \to K^*(892)\gamma$ to potential models. Among the radiative processes $B \to X_s\gamma, B \to K^*(892)\gamma$ and $B \to K_2^*(1430)\gamma$ exclusive branching ratios have been measured experimentally [1]:

$$\mathcal{B}(B \to K^*(892)\gamma) = (4.55 \pm 0.34) \times 10^{-5}, \tag{1}$$

$$\frac{\mathcal{B}(B \to K_2^*(1430)\gamma)}{\mathcal{B}(B \to K^*(892)\gamma)} = 0.39^{+0.15}_{-0.13} \tag{2}$$

and so has been the inclusive rate [2]

$$\mathcal{B}(B \to X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-5}. \tag{3}$$

Several methods have been employed to predict $B \to K^*\gamma$ decay rate: HQET [3,4], QCD sum rules [5–10], quark models [11–21], bound state resonances [22] and lattice QCD [23–26]. In this paper we follow the approach of [3,4] in which both b and s quarks are considered heavy. Although the s quark is not particularly heavy and very substantial corrections to the Isgur-Wise functions are to be anticipated, particularly at the symmetry point, it has been found that the use of heavy quark symmetry for the s quark has not been too bad [4], particularly in connection with the prediction of decay rates for $D \to K^{(*)} l \nu_l$, $D \to K l \nu_l$ from $B \to D^{(*)} l \nu$ [3].

In order to justify the use of heavy s-quark mass limit, we show that although this limit may be not so good for individual decay rates is quite justified for the ratios of decay rates. For this purpose we consider the overlap of the initial and final state s-wave functions:

$$I = \int d^3p \Phi_F \left[\mathbf{p} - \frac{m_{sp}}{M} \mathbf{K} \right] \Phi_I(\mathbf{p}) m_f \times \left[\frac{E + m_f}{EE'(E' + m_f)} \right]^{1/2}, \tag{4}$$

which can be determined without using the heavy quark limit. Here E and E' are the initial and final state energies,

$$K = \sqrt{\frac{m_{K^\star}}{m_B}} (m_B - m_{K^\star}) \tag{5}$$

and $M = m_f + m_{sp}$, where m_f and m_{sp} are masses of decaying and spectator quarks, respectively. In the heavy quark limit, the recoil correction (the second term in the argument of Φ_F) vanishes. We find that the ratio of the overlap integral for $B \to K_2^\star(1430)\gamma$ and that for $B \to K^\star(892)\gamma$ remains almost the same whether we calculate it using the actual integral or the one with $K = 0$, i.e., in the heavy quark limit. Actually this ratio comes out to be 0.678 for $K \neq 0$ and 0.689 for $K = 0$, if we choose Φ_F and Φ_I to be the Gaussian-momentum-state trial wave functions. Thus, it may be concluded that the ratios calculated in this paper are not sensitive to the heavy quark limit for "s." This conclusion agrees with the one obtained in [27].

In the heavy quark limit, the long distance effects are contained within unknown form factors whose precise defi-

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nition consistent with the covariant trace formalism [28–31] has now been clarified. We use the same nonrelativistic quark model for the wave functions of the light degree of freedom (LDF) as was used in [4] but employ the numerical approach of [32]. We use three different potentials: linear potential "$V=-4\alpha_s/3r+br+c$," screening confining potential "$V=(-4\alpha_s/3r+\sigma r)[(1-e^{-\mu r})/\mu r]$" [33], as suggested by lattice gauge theory, and heavy quark potential "$V=\sigma r-(8C_F/r)u(r)$" [34]. Our results for the linear potential are $(16.07\pm5.2)\%$ for $B\rightarrow K^*(892)\gamma$ and $0.45 \pm0.13)$ for $B\rightarrow K_2^*(1430)\gamma$ while those for the screening confining potential are $(19.75\pm5.2)\%$ and $(0.24\pm0.06)\%$ and for the heavy quark potential are $(11.18\pm4.6)\%$ and $(0.46\pm0.14)\%$, respectively. They are in good agreement with the recent experimental measurements made by CLEO [1] and at the present precision of both theoretical and experimental values, one cannot distinguish between the above potentials.

In Sec. II we present the theoretical framework for $B\rightarrow K^{**}\gamma$ decays. The Isgur-Scora-Grinstein-Wise (ISGW) model is described in Sec. III which also contains the procedure for finding the wave function of LDF using different potential models. The obtained results are summarized in the Conclusion.

II. THEORETICAL FRAMEWORK

For $B\rightarrow K^{**}\gamma$ decays the effective Hamiltonian is well known [32,35,36,37]:

$$H_{eff}=-\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*C_7(m_b)\mathcal{O}_7(m_b), \tag{6}$$

where

$$\mathcal{O}_7=\frac{e}{32\pi^2}F_{\mu\nu}[m_b\overline{s}\sigma^{\mu\nu}(1+\gamma_5)b+m_s\overline{s}\sigma^{\mu\nu}(1-\gamma_5)b]. \tag{7}$$

Matrix elements of bilinear currents of two heavy quarks $[J(q)=\overline{Q}TQ]$ are most conveniently evaluated within the framework of covarient trace formalism. Denoting $\omega=v'\cdot v$, where v and v' are the four velocities of the two mesons, we have

$$\langle\Psi'(v')|J(q)|\Psi(v)\rangle=\mathrm{Tr}[\overline{M}(v')\Gamma M(v)]\mathcal{M}(\omega),$$

where M' and M denote matrices describing states $\Psi'(v')$ and $\Psi(v)$, $\overline{M}=\gamma^0M^\dagger\gamma^0$, and $\mathcal{M}(\omega)$ represents the LDF. M, M', and $\mathcal{M}(\omega)$ can be found in [4,31] and using Eq. (7) we can write

$$\langle K^{**}\gamma|\mathcal{O}_7(m_b)|B\rangle = \frac{e}{16\pi^2}\eta_\mu q_\nu \mathrm{Tr}[\overline{M}'(v')\Omega^{\mu\nu}M(v)]\mathcal{M}(\omega), \tag{8}$$

where the factor $q_\nu=m_Bv_\nu-m_{K^{**}}v'_\nu$ comes from the derivative in the field strength $F_{\mu\nu}$ of Eq. (7), η_μ is the photon polarization vector and

$$\Omega^{\mu\nu}=m_B\sigma^{\mu\nu}(1+\gamma_5)+m_{K^{**}}\sigma^{\mu\nu}(1-\gamma_5). \tag{9}$$

Using the mass shell condition of the photon $(q^2=0)$ and polarization sums for spin-1 and spin-2 particles, the decay rates are calculated in [4,32]:

$$\Gamma(B\rightarrow K^*(892)\gamma) =\Omega|\xi_C(\omega)|^2\frac{1}{y}[(1-y)^3(1+y)^5(1+y^2)], \tag{10}$$

$$\Gamma(B\rightarrow K_1(1270)\gamma) =\Omega|\xi_E(\omega)|^2\frac{1}{y}[(1-y)^5(1+y)^3(1+y^2)], \tag{11}$$

$$\Gamma(B\rightarrow K_1(1400)\gamma) =\Omega|\xi_F(\omega)|^2\frac{1}{24y^3}[(1-y)^5(1+y)^7(1+y^2)], \tag{12}$$

$$\Gamma(B\rightarrow K_2^*(1430)\gamma) =\Omega|\xi_F(\omega)|^2\frac{1}{8y^3}[(1-y)^5(1+y)^7(1+y^2)], \tag{13}$$

$$\Gamma(B\rightarrow K^*(1680)\gamma) =\Omega|\xi_G(\omega)|^2\frac{1}{24y^3}[(1-y)^7(1+y)^5(1+y^2)], \tag{14}$$

$$\Gamma(B\rightarrow K_2(1580)\gamma) =\Omega|\xi_G(\omega)|^2\frac{1}{8y^3}[(1-y)^7(1+y)^5(1+y^2)], \tag{15}$$

$$\Gamma(B\rightarrow K^*(1410)\gamma) =\Omega|\xi_{C_2}(\omega)|^2\frac{1}{y}[(1-y)^3(1+y)^5(1+y^2)], \tag{16}$$

$$\Gamma(B\rightarrow K_1(1650)\gamma) =\Omega|\xi_{E_2}(\omega)|^2\frac{1}{y}[(1-y)^5(1+y)^3(1+y^2)], \tag{17}$$

where

$$y=\frac{m_{K^{**}}}{m_B}, \tag{18}$$

$$\Omega=\frac{\alpha}{128\pi^4}G_F^2m_B^5|V_{tb}|^2|V_{ts}|^2|C_7(m_B)|^2 \tag{19}$$

and the argument of the Isgur-Wise (IW) function is fixed by the mass shell condition of the photon ($q^2=0$):

$$\omega=\frac{1+y^2}{2y}. \tag{20}$$

III. POTENTIAL MODELS FOR THE ISGUR-WISE FUNCTIONS

Following [32] for the evaluation of IW form factors needed for the decay rates we assume that we can describe heavy-light mesons using some nonrelativistic potential models; the rest frame LDF wave function can then be written as

$$\phi^{(\alpha L)}_{j\lambda j}(x)=\sum_{m_L,m_s} R_{\alpha L}(r)Y_{Lm_L}(\Omega)\chi_{m_s} \times\left\langle L,m_L;\frac{1}{2},m_s\middle| j,\lambda_j;L,\frac{1}{2}\right\rangle, \tag{21}$$

where χ_s represent the rest frame spinors normalized to one, $\chi^{\dagger}_{m_s'}\chi_{m_s}=\delta_{m_s',m_s}$ and α represents all other quantum numbers. In [32] the following expressions for the form factors are obtained:

$$\xi_C(\omega)=\frac{2}{\omega+1}\langle j_0(ar)\rangle_{00}, \quad 0^-_{1/2}\rightarrow(0^-_{1/2},1^-_{1/2}), \tag{22}$$

$$\xi_E(\omega)=\frac{2}{\sqrt{\omega^2-1}}\langle j_1(ar)\rangle_{10}, \quad 0^-_{1/2}\rightarrow(0^+_{1/2},1^+_{1/2}), \tag{23}$$

$$\xi_F(\omega)=\sqrt{\frac{3}{\omega^2-1}}\frac{2}{\omega+1}\langle j_1(ar)\rangle_{10}, \quad 0^-_{1/2}\rightarrow(1^+_{3/2},2^+_{3/2}), \tag{24}$$

$$\xi_G(\omega)=\frac{2\sqrt{3}}{\omega^2-1}\langle j_2(ar)\rangle_{20}, \quad 0^-_{1/2}\rightarrow(1^-_{3/2},2^-_{3/2}), \tag{25}$$

where, denoting the energy of LDF as E_q,

$$a=(E_q+E_{\bar{q}'})\sqrt{\frac{\omega-1}{\omega+1}} \tag{26}$$

and

$$\langle F(ar)\rangle^{\alpha'\alpha}_{L'L}=\int r^2drR^*_{\alpha'L'}(r)R_{\alpha L}(r)F(ar). \tag{27}$$

To find the form factors we use the method of [32], i.e., solve the Schrödinger equation numerically. We use three different potentials.

The linear potential:

$$V=\frac{-4\alpha_s}{3r}+br+c. \tag{28}$$

The screening confining potential [33]:

$$V=\left(\frac{-4\alpha_s}{3r}+\sigma r\right)\frac{1-e^{-\mu r}}{\mu r}. \tag{29}$$

the heavy quark potential [34]:

$$V=\sigma r-\frac{8C_F}{r}u(r). \tag{30}$$

For the linear potential we use the same parameters as determined in [32], namely $b=0.18$ GeV^2, α_s in the range 0.37 to 0.48 and c in the range 0.83 to -0.90 GeV. In [32], these values were obtained by fixing [38] $b=0.18$ GeV^2, and varying α_s and c for a given value of $m_{u,d}$ and m_s (respectively in the ranges 0.30–0.35 GeV and 0.5–0.6 GeV), until a good description of the spin averaged spectra of *K*-meson states is obtained. These parameters are in good agreement with the original ISGW values [38] ($\alpha_s=0.5$ and $c=-0.84$ GeV). For the screening confining potential [33], $\sigma=0.18\pm0.02$ GeV^2 and $\mu^{-1}=0.8\pm0.2$ fm, while for the heavy quark potential, σ is as given above, $C_F=(N_c^2-1)/2N_c$, and [$a(q^2)$ is defined in [34], $k=\sigma/2\pi C_F$]

$$u(r)=\int_0^{\infty}\frac{dq}{q}\left(a(q^2)-\frac{k}{q^2}\right)\sin(q\cdot r), \tag{31}$$

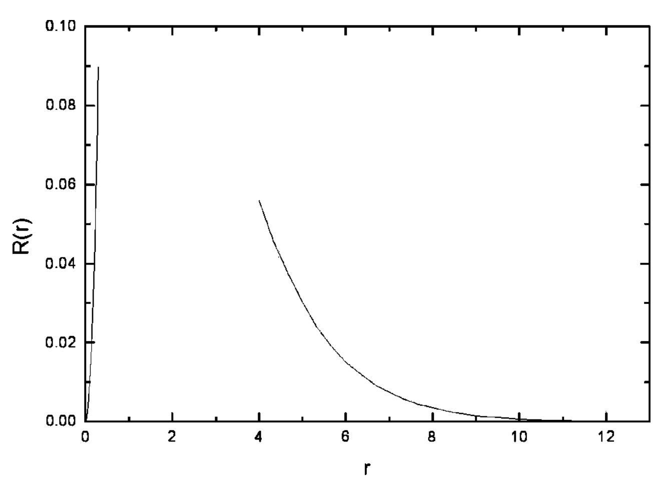

FIG. 1. Low *r* and high *r* (asymptotic) behavior of the LDF wave function.

TABLE I. Ratios $\equiv BR(B\to K^{\star\star}\gamma)/BR(B\to K^{\star}(892)\gamma)$.

Meson	J^P	Linear Pot.	Scr. Pot.	Heavy Quark	Ref. [32]	Experimental value
$K^\star$	0^+		Forbidden			
K_1	1^+	0.29 ± 0.08	0.42 ± 0.10	0.15 ± 0.04	0.26 ± 0.07	
K_1	1^+	0.15 ± 0.04	0.19 ± 0.05	0.49 ± 0.15	0.13 ± 0.03	
$K_2^\star$	2^+	0.45 ± 0.13	0.24 ± 0.06	0.46 ± 0.14	0.37 ± 0.10	$0.39^{+0.15}_{-0.13}$
$K^\star$	1^-	0.03 ± 0.01	0.19 ± 0.05	0.12 ± 0.04	0.03 ± 0.01	
K_2	2^-	0.12 ± 0.03	0.04 ± 0.01	0.16 ± 0.05	0.10 ± 0.03	
K	0^-		Forbidden			
$K^\star$	1^-	0.34 ± 0.10	0.32 ± 0.08	0.28 ± 0.08	0.24 ± 0.07	
$K_0^\star$	0^+		Forbidden			
K_1	1^+	0.11 ± 0.03	0.16 ± 0.04	0.18 ± 0.05	0.10 ± 0.03	

which is calculated numerically at $r\geqslant 0.01$ fm and represented in the MATHEMATICA file.[1] The short distance behavior of the potential is purely peturbative, so that at $r\leqslant 0.01$ fm we can put

$$V(r)=-C_F\frac{\overline{\alpha_v}(1/r^2)}{r}, \tag{32}$$

where the value of the running coupling constant $\overline{\alpha_v}(1/r^2)$ at $r_s=0.01$ fm is $\overline{\alpha_v}(1/r_s^2)=0.22213$.

As in [4] the definition of the LDF energy for a K^{**} meson, proposed to account for the fact that s mesons are not particulaly heavy, is

$$E_{\bar{q}}=\frac{m_{K^{**}}\times m_{u,d}}{m_s+m_{u,d}}. \tag{33}$$

Another definition which is consistent with heavy quark symmetry is

$$E_{\bar{q}}=m_{K^{**}}-m_s. \tag{34}$$

These two definitions are not equivalent in the heavy quark limit, so we have done all calculations employing both of these two definitions and at the end we have quoted the broadest possible range of the results obtained. Finally, $E_{\bar{q}}$ for the B meson has been taken to be the same as for the K^* meson, consistent with heavy quark symmetry. It turns out that this is actually a very reasonable assumption. To find the size of a meson, which we need for the evaluation of the integral in Eq. (27), we investigate for the asymptotic behavior of the Schrödinger equation for a particular potential model. As an example we display the asymptotic behavior of the LDF wave function for a linear potential model (for $l=1$) in Fig. 1.

[1]In the format of note book at the site http://www.ihep.su/~kiselev/Potential.nb.

IV. CONCLUSION

In Table I we present our results for the ratio $R=\Gamma(B\to K^{\star\star}\gamma)/\Gamma(B\to X_s\gamma)$ for various K meson states; the inclusive branching ratio is usually taken to be the QCD improved quark decay rate for B which can be written as [35,37]

$$\Gamma(B\to X_s\gamma)=4\Omega\left(1-\frac{m_s^2}{m_b^2}\right)^3\left(1+\frac{m_s^2}{m_b^2}\right) \tag{35}$$

giving the prediction for $BR(b\to s\gamma)$ to be $(2.8\pm0.8)\times10^{-4}$ [32], where the uncertainty is due to the choice of the QCD scale.

We find that the radiative decays of B into K meson states saturate 30% to 50% the inclusive decay rate. We cannot reach more quantitative conclusions due to errors involved in theoretical estimates, due to which we also cannot at present distinguish between the potential models used.

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